English 467: Journalism
Professor Jill Abramson

By submitting this essay, I attest that it is my own work, completed in accordance with University regulations.—Jeremy Lent

Prove It

by Jeremy Lent

Top U.S. college math students have been wrestling with the six-hour, twelve-question William Lowell Putnam Mathematical Competition nearly every December since 1938. The past few decades have seen 2,000, then 3,000, and now 4,000 competitors per year. Each question is worth ten points, for a total possible score of 120. The median score is one or two points. Do the math: It's a hard test.

Each year's top five test-takers, who usually score around 100, are crowned as Putnam fellows and receive $2,500, along with eternal bragging rights in the math community. There's also an inter-college competition. Each participating school designates three students—usually the ones expected to score highest—whose individual rankings are combined to give a team score. The top five teams get bragging rights as well, at least until the next year's exam.

The list of individual and team winners isn't terribly surprising. Harvard's team has placed in the top five 55 times in the last 72 years (19 times in the last 20), alongside MIT's 39 and Princeton's 28. Harvard has also spawned 98 top-five individuals since 1938, with nearly one per year for the past 20 years.

When Xiaosheng Mu, a Yale freshman from China, settled into the test last December, Yale hadn't produced a top-five team since 1991, or a top-five individual since 1989. Mu couldn't help with Yale's team-win drought: Freshmen never make the Yale team, since members are chosen based on at least one previous Putnam performance. But Mu did something right in the ensuing six hours. Actually, more like 11 or 12 things right.

On March 23 this year, the Mathematical Association of America named Mu as one of the five 2009-2010 Putnam fellows. Once word got out, Yale's student newspaper and press office made good use of the bragging rights. However, none of the articles or press releases mentioned that Yale's team had scored about thirtieth. Out of a record 439 teams in 2009, that was respectable, but not anything that will keep Harvard's team up at night.

Aside from the two-dozen other Yalies who sat for the Putnam along with Mu, very few students at Yale likely have any idea (or a desire to have one) what the upper echelon of mathematics looks like. For the last ten years, Yale has graduated an average of 11 math majors each year (out of about 1,250 students per graduating class), down from an average of 22 majors between 1976 and 1998. The number of Yalies enrolled in math classes has dropped as well, from around 1,900 students in 1978 to about 1,500 today. Over half of those students took one of the introductory calculus or linear algebra classes, prerequisites for many other majors besides math. Unless Yale students stick around for an upper-level math class—which few do—they never get a taste of the kinds of problems on the Putnam, whose answers (if you ever reach them) are page-long proofs, and not just a few numbers.

Is it the small and shrinking pool of upper-level math students that's kept Yale far behind its Putnam rivals? That's probably part of the picture. Harvard's math department has shepherded through about 250 majors annually in recent years, and MIT has seen around 350. (Yale's graduating classes are smaller than Harvard's, and larger than MIT's, by about 250 students). According to Noam Elkies, a math professor at Harvard since 1990 and a Putnam fellow himself, somewhere between 50 and 100 Harvard students take the Putnam each year, compared to Yale's 20 or 25.

But as Salah Ahmed, a current math major at Yale, pointed out, Yale would need only a few of the world's top math students to win big in the Putnam. "If all the IMO [International

Mathematical Olympiad] gold medal winners from China apply to Yale, and Yale accepts them all, then Yale will have a strong Putnam team. It's as easy as that," Ahmed said, noting that Xiaosheng Mu won gold for China in the 2008 IMO, a global high school competition to which over 100 countries send six of their brightest students.

Why doesn't Yale attract more students from the high school competition circuit? Ahmed, who nearly made the IMO team in his native Pakistan, thinks that part of the problem is tradition. Harvard and MIT, unlike Yale, are known as Putnam powerhouses and so draw the upper crust of the teenage math community. It probably also doesn't help that Harvard and MIT jointly run one of the biggest annual math competitions for U.S. high schoolers, the Harvard-MIT Mathematics Tournament. Maybe Putnam success is all about name recognition.

Roger Howe, who has taught math at Yale since 1974, claims not to be miffed by the dominance of his neighbors to the north. He compares the growing scrutiny over Putnam results to the country's "unhealthy obsession with professional sports," and he doesn't see Putnam training as a priority for Yale's department. "Becoming a good test-taker is a good goal, but I think there should be a balance between that and learning the math curriculum," Howe said.

Then again, Howe was a Putnam fellow in 1964, and according to Max Engelstein, a senior math major at Yale, he's very careful when helping to select Yale's Putnam team each year.

Yale has another disadvantage that several math majors pointed to, and that's a deficiency in what they call "math culture." When Engelstein, who scored in the top 200 on the Putnam last year, arrived at Yale in 2006, he found far fewer math enthusiasts than at Stuyvesant High School, a math and science engine in New York City. In fact, Yale didn't even have an undergraduate math club (there was one that died out in 2004), leaving a conspicuous gap next to the large and active groups for physics and computer science.

At the end of Engelstein's freshman year, one of his math professors asked him to revive the Yale Undergraduate Math Society. For the past three years, the Society has held weekly Putnam practice sessions in the fall, along with some talks by math professors and students on topics like fractals (incessantly repeating geometrical figures) and "math puzzles."

But Mu, the freshman Putnam fellow, didn't go to many of the practice sessions, preferring to train on his own. And he described the few Math Society lectures he attended this year as "introducing fun math to students who don't want to get that deep into it."

Engelstein admits that Yale has nowhere near the math buzz that Harvard enjoys. His friends from Harvard tell him that most of Harvard's math majors hang out in the math department building, work on homework together and, importantly, keep food lying around to lure anyone who has a math question.

With a quick glance at Harvard's lively math department website, you can see why students might be drawn to the sleek, couch- and computer-filled Harvard Science Center, which houses the math, statistics, and history of science departments, along with a full-service cafeteria and the Math Question Center, staffed five nights a week by undergraduate course assistants. Harvard also boasts the Harvard College Mathematics Review, an undergraduate journal that accepts submissions from students and faculty around the country. The weekly Mathematics Colloquium meets in one of Harvard's private dining rooms on Tuesday nights, while the more casual Math Club hosts, among other events, a pie-eating party on Pi Day (March 14, or 3.14).

Judging by websites alone, Yale's math department just doesn't measure up. As of early May 2010, Yale's math department website still had no mention of Mu's Putnam win—only an outdated announcement about the upcoming 2009 Putnam exam. Harvard's site has a full page on Putnam results from the past ten years, along with pictures of each year's Harvard team and a "Putnam practice problem of the day," which is, true to its word, updated daily.

Harvard's math website is filled with pictures of the Science Center, but Yale's site offers no peek at the Yale math department's home, Leet Oliver Memorial Hall, a relic from 1908 that was built originally for English, economics, and history classes. In the 1960s, the University budgeted for a brand new mathematics building, and even held a high-profile design competition won by architect Robert Venturi. But Yale fell into financial trouble soon after, and the new math center never materialized.

Leet Oliver Hall is not as musty as you'd expect for a century-old building, but it's perhaps more somber than the math that goes on inside. The echoing marble floors make the building feel empty even when it's filled with students. Very little talking goes on in the hallways between the dozen classrooms.

At the top of the entrance staircase is a six-foot painting—no title or artist's name is provided—of an ascending mesh of arches, densely packed until the very top, where a few turrets poke out into a muddy sky. First-year calculus students, filing out from their symbol-filled lectures, are undoubtedly reminded of the Tower of Babel.

To the painting's left is a map of Yale, dated 1961, alongside a three-shelved trophy case that holds only four trophies: one for Yale's fifth place finish in the Putnam in 1969, the other three for honorable mentions in 1970, '71 and '77. The rest of the shelf space is filled with white plaster models of conic sections and other bulbous geometric shapes.

Down the hall is another glass case, displaying two dozen mathematically titled books and a faded yellow sign that proclaims, "Yale Ph.D.s Publish!" The exclamation point makes it seem as though someone was surprised at the fact—although to be sure, Yale garnered tenth place in the most recent U.S. News and World Report rankings for math graduate programs. (MIT was first, and

Harvard was second.)

About half the books in the case were published in the ‘60s or ‘70s, although a few books, like “Lectures on Automorphic L-Functions” (2004), are from this millennium. But outdating all these books is a withered postcard, inexplicably wedged outside the glass in the lower left-hand corner of the case frame. It was sent on September 26, 1941 to Mrs. Knapp at 12 Hillhouse Avenue (the address of Leet Oliver Hall). The scribbles on the postcard say something about the fresh air in Canada, but the initials of the signatory, “H.L.S.,” are unfamiliar to Paul Lukasiewicz, the librarian who’s kept watch over Yale’s math book collection, housed on the third floor of Leet Oliver, since 1966.

Lukasiewicz never studied math beyond high school but now keeps up daily with who’s publishing what in the world of academic mathematics. He sees a steady stream of visitors at the beginning of each semester, when students are scavenging for textbooks. But after the first few weeks, the crowds thin out, and just about the only noises come from the shuffling of graduate student feet.

Lukasiewicz remembers the various iterations of the Yale Math Society over the years, as well as the math graduate student softball team that he captained until it dissolved a few years after his arrival. Now, his soft voice and smiling, double-chinned face give the impression that he rarely has to shush anyone.

Many math students likely prefer silence while they work, but there’s no comfortable place in Leet Oliver for students to work, either together or by themselves. Engelstein says that this lack of a home base is a major barrier to building a math community like Harvard’s.

Despite the Hollywood image of the solitary math genius scrawling equations on a blackboard in an empty lecture hall, being part of an academic community can be crucial for budding mathematicians. Out of the seven current or recent math majors interviewed for this article, all had had plenty of social math interaction in high school, either by training and competing with a math team or by attending summer math camps, which are part of a recently growing push to introduce promising high schoolers to advanced math.

Shira Helft, who captained her high school math team in Port Washinton, NY, went to math camp after her sophomore year of high school, then topped it off with theoretical computer science research at Boston University the following summer. But during her first year of math classes at Yale, Helft felt that she was "falling out of love with math."

"Outside of my math classes, I was in a community of people who didn't believe in the power of math. Most Yale students simply don't believe that higher-level math is important for their education," Helft said.

The next year, Helft helped Engelstein get the Math Society going, but when she continued to miss her high school math community, she started looking for math inspiration beyond Yale's walls. In the fall of 2009, she left Yale for a semester to enroll in the Budapest Semesters in Mathematics, a program that annually draws about 50 top U.S. and Canadian math students to be taught (in English) by instructors from Hungary's Mathematical Institute. Helft loved the camaraderie.

"We developed personal relationships with the professors, we worked in pairs to solve problems in class, we always had games and competitions, and all the classes were in one hallway, so you couldn't avoid getting excited about math," Helft said.

Math mentorship and community were important for Mu, the freshman Putnam fellow, who studied independently for Chinese math competitions under the guidance of a math teacher who taught him throughout his six years of middle and high school. Mu's high school was a well-known feeder for China's Mathematical Olympiad team, and many of his classmates shared his math ambitions, if not his talent.

"At my high school, we always discussed problems with each other. Here at Yale, you tend to do things individually," Mu said. (Mu was rejected from Harvard, and he chose Yale because it was the closest, out of the schools that accepted him, to the vibrant math community in Cambridge, MA.)

The biggest reason why Yale falls so far behind Harvard and MIT on the Putnam may be a chicken-and-egg problem (which belongs in the province of biologists, not mathematicians): Yale doesn't attract the kinds of students who would build up a strong math community because Yale doesn't have those kinds of students in the first place.

And Leet Oliver Memorial Hall can't be helping the situation.

It's true that hundreds of Yalies each semester take a 100-level calculus class, but enrollments drop exponentially as the course numbers rise. Last fall, "Linear Algebra and Matrix Theory" (Math 225), drew nine undergrads, while "Modern Algebra" (Math 380) had just six.

There's a fairly simple explanation for this trend, you might think: Math classes are hard, and upper-level math classes are harder. As Helft points out, most of the people taking 200-level classes are the ones who went to competitions and math camps in high school. And the intro calculus classes don't often inspire students to take any more math than they have to, said Yen Duong, a math and philosophy major.

"There are very few things that could scare off more people from math at Yale than the intro calculus classes. And they hardly prepare you for higher-level math classes," Duong said. The consistently poor evaluations that Yale students give of many of these intro classes—taught mostly by graduate students— back up Duong's impression.

So what if you weren't a math Olympian in high school but are looking to take an upper-level math class at Yale? This is where I come into the story.

I wanted to be a math major when I came to Yale as a freshman in 2007. At my high school in Montgomery County, MD, I took AP Calculus in my junior year, won “Best Math Student” in senior year, and I was even awarded a Rensselaer Polytechnic Institute math and science medal based on my teachers’ nominations. So when I surveyed Yale’s math offerings, I decided on Math 230, “Vector Calculus and Linear Algebra.” The class was described in the course listings as a warp-speed primer for prospective math majors who had already been introduced to calculus, thank you very much.

The course evaluations that I read before getting to campus were foreboding. “Math 230 is a rite of passage . . . you will feel hopelessly lost at pretty much every point during the course. Find a group of friends to work with early; homework is impossible otherwise,” wrote one student. (Yale’s evaluation system is anonymous.) The scent of challenge excited me. I was used to hearing classmates complain that math was “impossible,” and I was used to proving them wrong.

About 70 other freshmen showed up to the first Math 230 class, but thanks in part to Yale’s “shopping period”—when students get to sit in on classes for a week before committing to a course schedule—our numbers were down to 30 by the second week. However, the material in those first weeks was innocent enough: axioms of arithmetic, the definition of a function, a little vector addition. The homework problems were also not that difficult, although they were unlike anything I had seen in high school. We were asked to “prove” various theorems, rather than write out a few formulas and do some calculator tricks, as I had done and excelled at throughout high school.

As the fall days got shorter, the proofs grew longer. My assignments stretched out for several notebook pages whose lines were filled with complete, albeit symbol-filled, sentences. And each lecture became a dactylic marathon of writing down all the proofs that the professor, a stocky postdoc from Lebanon, copied from his notes onto the blackboard. Like several of my classmates, I perceived no other choice than to be a stenographer, since there was no way I was going to understand those proofs in real-time.

All the proofs were ostensibly logical, but they started making less and less sense to me. I tried working on homework problems with some of my similarly confused classmates, and I visited the graduate student math tutors. But for each seven-hour assignment I finished off, I heard a rumor about some kid finishing in two hours, or one hour, or half an hour.

I found out pretty quickly that most of the freshmen in Math 230 had been doing proofs in high school, often at competitions and math camps. Meanwhile, I had been chugging away happily at trigonometry and calculus problems that had only one right answer—the only kind of math problem that most high schoolers and college students will ever see. (There are many, sometimes infinite, ways to write a correct proof for a given theorem.) I was great at those problems, and I expected college math to be more of the same.

In fact, I was mathematically justified to expect that. The only proofs required of me by the math curriculum in Montgomery County Public Schools, one of the country's highest-achieving school districts, were short blurbs about congruent and similar triangles in middle school geometry. Sure, there were some proofs in the appendix of my junior year calculus textbook, but we never had to look at them, and I assumed that most mathematicians don't bother with them either.

In Math 230, I learned that mathematicians bother with nothing but the proofs, and the same is expected for undergraduates who venture beyond the intro courses. Proof-based homework assignments are like very complicated Jeopardy problems. The theorem (or "answer") is given, and student has to show how to arrive at the theorem using only basic assumptions (called axioms) and foolproof steps of logical reasoning. Which axioms to use, and which steps to take, is often far from obvious.

Professional mathematicians work at coming up with these theorems in the first place, a process that can take years and dozens of pages. But they don't prove new theorems just to give their students more homework problems. Proofs are the engine of mathematics, the tools that Euclid, Descartes, Newton, and Einstein used to make their far-reaching contributions. Anyone with a bit of mathematical vocabulary can make a conjecture—for instance, that there exist infinitely many prime numbers, or that the area of a circle is πr^2—but it takes a proof-wielding mathematician to show whether or not that conjecture is right, whether or not scientists can use the conjecture in their computations, and whether or not mathematicians can use it to prove more theorems.

There's an even more practical reason why mathematicians spend most of their time on proofs: The rest of the mathematical labor market is being outsourced to technology. Between modern graphing calculators and an online site called Wolfram Mathematica, computer chips can do the bulk of the work for just about any problem that I solved in AP Calculus. In fact, computers have even started doing proofs. The first famous "computer-assisted proof" surfaced in 1976. That year, two mathematicians from the University of Illinois wrote a computer program to check all 1,936 contingencies that needed to be checked to prove the four-color theorem. This theorem states that for any map (a division of a 2-D plane into multiple regions by straight or curved lines), there is a coloring scheme, using no more than four colors, such that no two adjacent regions are colored the same. The four-color theorem was first conjectured in 1852, but the sheer immensity of work needed for confirmation stymied efforts at a proof by unassisted mortal hands.

Since 1976, several more important conjectures have emerged from computerized proof programs. For now, mathematicians still have the proof-writing market mostly cornered, although that doesn't guarantee that math departments of the future won't be staffed by computers. In any case, it's clear that number-crunching high school math classes are no longer much of a job qualification.

I did need some of my high school algebra and calculus skills to do the proofs in Math 230, but since a proof hadn't crossed my desk since middle school, I felt helplessly behind most of my classmates. If I had gone to high school four decades earlier, things might have been different. When Linda Loomis, my high school calculus teacher, started as a ninth grader at a D.C. public school in 1964, she was doing proofs from day one. In algebra, her class started off by proving that the square root of two is an irrational number. In geometry, they did "exactly what Euclid did" in his proof-filled "Elements." AP Calculus, in the absence of calculators, was all about proofs in the 1960s. In 2006, the year I took calculus, the AP exam had one proof question, alongside nearly a hundred one-right-answer problems.

Loomis's high school math classes were an outgrowth of "New Math," a nationwide curricular movement launched soon after Sputnik that stressed abstract concepts, with an eye towards training future scientists and engineers. Loomis went on to get a degree in proof-based math from American University in 1972, and she started teaching high school math in 1977. But by then, the pendulum was swinging back towards number-crunching and real-world applications, a trend that solidified in the late 1980s, Loomis recalls, as calculator companies such as Texas Instruments began colonizing classrooms.

In any case, New Math wasn't going over so well. Although Loomis had the right credentials, many of her fellow teachers hadn't been trained in the set theory or base-six number systems that they were expected to teach. Besides, the new material was soaring over most students' heads.

"Finally, people realized that not everyone's going to be a rocket scientist," Loomis recalled.

As the Cold War subsided, college-readiness replaced battle-readiness as the educational priority. This meant getting as many students as possible ready for college. And if that was going to happen, more students needed to be taking higher-level math in high school. Since many high school students aren't ready for proofs, Montgomery County's school officials simply took the proofs out of the higher-level classes, Loomis explained.

But I suspect that I would have found Math 230 more manageable if Montgomery County had stuck with the proofs. As it was, I felt overwhelmed after the first two months of my freshman year, and I dropped down to a 100-level calculus class. (That's one of the few mid-semester class changes that Yale allows. As I found out, many students before me had taken the same route.) I happily found myself back in the world of number-crunching, one-right-answer math. Calculator in hand, I finished out the semester with an A.

I'm not the only one who backed out. Ahmed, the math major from Pakistan, started Math 230 with me and dropped it a week before I did. But even more than I, he seemed to have the right credentials to take an upper-level math class in freshman year. Aside from nearly earning a spot on Pakistan's Mathematical Olympiad team, Ahmed scored higher than any Pakistani student in 2005 when he took the Cambridge International Examination O-Level math test, given to many of Pakistan's top tenth graders. Two years later, he got the highest A-Level score in his hometown, Karachi, Pakistan's largest city.

The O-level and A-level exams weren't proof-heavy, Ahmed said, but he had done plenty of proofs at summer math camps and at St. Patrick's High School in Karachi. He recalls that his math teachers in Pakistan focused carefully on proof-writing skills, letting students work through proofs together in class and waxing philosophic about what makes certain proofs "beautiful" and others "ugly."

But Math 230 might as well have been a different subject, since Ahmed could hardly grasp what was going on.

"The professor would just write down all the proofs on the blackboard. But all those proofs are on Wikipedia, and I can read them myself. I'm literate enough to do that. I want the professor to go through the proof with me, because proofs are not always obvious. You can read through them, but you won't necessarily understand them."

So perhaps it wasn't the proofs themselves but the teaching style that gradually rendered Math 230 incomprehensible to me. A different professor, one who taught like Ahmed's teachers in Pakistan, might have made me feel more at home mathematically. But several Yale math majors explained that my professor's teaching style is common in upper-level courses.

"We get a whirlwind of results. We prove theorems for the sake of it, without talking about the motivation, the reason that the theorems are important. And you sometimes end up losing the elegance and beauty of math," said Edward Chang, a senior majoring in math and philosophy.

Duong gave a similar message. "I always feel like I'm learning almost everything by myself, from the textbook. When I study with other students, it helps. But if I were more engaged in class, that would really help."

Engelstein, the Math Society founder, concedes that he doesn't pick up much during lectures, and that he really understands the math only when he starts doing proofs for homework. Engelstein, Duong, Chang, and Ahmed (who returned to math classes with more determination the semester after he dropped out of Math 230) all said they were able to stick with math at Yale because they had already developed solid proof-writing skills in high school.

Evidently, there were two factors conspiring against me in Math 230: I had virtually no exposure to proofs in high school, and I had a professor who, like most, taught under the assumption that anyone taking a 200-level course had exposure to proofs. The prove-and-run teaching style was clearly adequate for some of my Math 230 classmates, like the ones who finished the homework at light-speed. But I wasn't nearly ready for the "whirlwind of results." And as I found out after dropping Math 230, the proof-free intro calculus classes wouldn't have prepared me any better for Math 230 than the classes I took in high school.

So on behalf of Yale students like me who don't get proofs in high school and who can't catch all the proofs thrown at them in upper-level math classes, I began to ask: Where are we supposed to learn how to do this stuff?

Unlike Yale, Union College in Schenectady, NY has offered a course specifically designed to introduce proof-writing skills to the mass of students who emerge calculator-addicted from high school. Maybe if Yale had a class like that, and if I had been directed towards it, I could have avoided my Math 230 trauma.

But most students who took the class at Union College struggled to apply their general proof skills to a content-specific upper-level math class, said Michael Frame, who taught at Union before moving to Yale in 2000 to oversee the introductory calculus classes. A better method, Frame said, is to learn proofs at a slow pace in a lower-level subject like algebra I or geometry, as Loomis and many of my Math 230 classmates had done.

Then again, as Loomis pointed out, many students aren't ready for proofs when they take algebra I or geometry in high school. And perhaps it's unreasonable to expect Yale's math department to offer proof-based versions of high school subjects. In any case, Frame doesn't think that any single course could succeed in getting every interested Yale freshman up to speed on proofs.

"If there were an obvious solution, I'm sure that we as mathematicians would have found it. But just like there are as many Buddhisms as there are Buddhists, there are probably as many ways to learn proof-writing as there are math students," Frame said

Some of the math majors I spoke with admitted that the proof barrage in upper-level classes is tough to handle. But they also said that professors wouldn't cover nearly as much material if they followed the teaching methods used at summer math camps, where most of these math majors got their first taste of proof.

Engelstein spent six weeks at a math camp at Hampshire College after his sophomore year of high school, and he describes those weeks as his mathematical coming-of-age. He spent six hours a day in classes led by a professor and two college students, who proposed a few definitions, then slowly explained proof techniques that Engelstein and his classmates used to prove a substantial bulk of modern number theory. Importantly, Engelstein says, the students weren't told which theorems to prove. They had to arrive at the results using only their new proof skills and their own intuitions.

"I remember proving these famous theorems by myself at one in the morning," says Engelstein, who thinks that Yale should offer a summer-camp-style seminar (although with fewer classroom hours) for freshman who are "wondering whether to be math majors but aren't sure yet, who haven't seen many proofs before."

Would math professors really put up with giving seminars instead of lectures? They do at St. John's College, a 500-student liberal arts school with campuses in Annapolis, MD and Santa Fe, NM. The St. John's curriculum centers around the "Great Books" of every liberal arts discipline, mathematics included. According to the college website, undergraduates study math for all four years, in tutorials of 12 to 15 students. They read works by Euclid, Descartes, Newton, and Einstein, then do proofs and discuss the big-picture questions all at the same time.[1]

This approach made a convert out of Jennifer Gavin, a high school English teacher in Westchester County, NY who has taken graduate classes at St. John's Annapolis campus for the past

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[1]Sources and Notes

St. John's survey: http://www.stjohnscollege.edu/resources/career.pdf two summers. Gavin diagnoses herself with a "math disability," at least when it comes to the number-crunching math she faced in high school and during her freshman year at Mary Washington College, before settling into an English major. But when she grudgingly began the St. John's graduate math segment (required for all graduate students, and structured like the undergraduate tutorials), she was surprised at how much she understood.

"It was still math, but it wasn't in this harshly competitive, right-or-wrong atmosphere that I always associated with math. It was just a group of people genuinely interested in finding out what the truth was," Gavin said.

Each day, Gavin's tutorial started with a discussion of some philosophy readings about the nature of numbers or the true essence of geometrical points. But when it came time to look at the proofs, her instructor didn't go the blackboard—instead, students did. Without notes, Gavin or one of her classmates would write up some of Euclid's axioms, then try to reach the same theorems Euclid did, getting pointers from the other students along the way.

"Sometimes it took us two hours to get through four proofs, but by the end you really knew it. I could really see the math, which is something that never happened to me in high school," Gavin said. She was even inspired to write a post about geometry on her otherwise-literary online blog, "Logos."

To be sure, St. John's core mission as a Great Books school isn't to train the next generation of research mathematicians (although in a 2000 survey, 9.4% of St. John's alumni reported having a career in computers, science, or math). A slow-moving math seminar with philosophical overtones doesn't quite fit the Yale math department's mission of "doing a good job for students who want to take math classes" but "not necessarily swelling the number of math students," according to Mikhail Kapranov, the current chair of Yale's math department. "Doing a good job," Kapranov said, includes covering lots of material in upper-level classes to prepare math majors for graduate study. Covering lots of material, as several math majors pointed out, requires fast-paced lectures, with little time to pause and reflect on the proofs, let alone invite students up to blackboard.

But is it worth covering so many proofs so quickly, if even the math majors struggle to keep up in lecture? And why do so many professors at Yale use a proof-by-fire lecture style? Part of the answer, it turns out, is graduate school.

"We didn't specifically talk about how to teach students in more advanced undergraduate courses," said Manish Patnaik, a graduate student in Yale's math department from 2001 to 2008 and now a postdoc at Harvard, recalling the one-semester teaching seminar that he took at Yale. At least at Yale and Harvard, Patnaik said, graduate students are required to teach only the intro-level calculus classes, which don't cover any proofs. As a result, graduate students get teacher training focused only on those courses. However, Patnaik hesitated to say that graduate students should get specific training about how to teach proofs in upper-level courses.

"Teaching proofs is maybe slightly easier for graduate students because that's what they have more exposure to. You remember your undergraduate proof classes and your graduate classes, which are all proof-based, whereas intro calculus is a more remote thing for most graduate students," Patnaik said.

At least at Harvard and Yale, the only model that graduate students get for their own future proof-based teaching comes from their proof-based classes in college and graduate school. Since many of those classes are taught as rapid-fire lectures rather than St. John's-style seminars, it's not surprising that new faculty members, like my Math 230 professor, take the lecture approach.

Kapranov, the department chair, defends the lecture-style teaching and the lack of emphasis on proofs in Yale's teacher training program. "Too much attention to the issue of teaching proofs is not constructive. Proof is simply a manifestation of common sense. The more it's built up as something transcendental, something that teachers and students need special training for, the more difficult it becomes," he said.

However, if Kapranov and the other math professors want to do something to improve Yale's flaccid undergraduate math culture—short of recruiting Mathematical Olympiad gold medalists—they might need to pay more attention to the issue of teaching proofs to the uninitiated like me. And who knows? A freshman seminar like the one Engelstein suggested, or a slower seminar-style approach in the upper-level classes, could increase Yale's pool of qualified competitors for the Putnam exam, which is entirely proof-based.

But if Yale's math department isn't going to change its ways anytime soon, high schools might be the best last hope for the future of Yale's undergraduate math community. If school officials reinstated the proofs that Loomis did when she was in high school, Yale might see a bigger crowd pushing through Math 230.

In fact, wonders Edward Chang, why not introduce proofs before high school? "There's no reason why an elementary school student can't understand elementary number theory or elementary set theory," Chang said, citing the short definitions and simple proofs that he began with at summer math camp.

But don't expect a "New New Math" movement anytime soon. Since September 2009, 48 states plus the District of Columbia have signed on to the Common Core State Standards for Mathematics, pledging to use the standards as a basis for curriculum reform. Almost every one of the 239 high school math standards in the current Common Core draft begins with the words "solve," "graph," or "use." Only ten of the standards include the words "proof" or "prove."[2]

The Common Core Standards are a blueprint for more number crunching in higher-level high school courses. However, Loomis is careful to look on the bright side.

[2] Common Core State Standards: http://www.corestandards.org/the-standards/mathematics

"We're now trying to make sure that more kids are exposed to higher-level math. We've succeeded at that, and that's a good thing," Loomis said. Also, she stressed that all the proof-based classes she took in high school were reserved for just the top few dozen math students. The rest of her classmates, she said, were "weeded out" of the higher-level math track and, as a result, had fewer qualifications for college admission. Future Yale applicants can probably handle a few proofs, but plenty of high school students struggle with the basics. The Common Core Standards perhaps rightly focus on the basic mathematical literacy needed for college and for careers that aren't in a math department.

As Michael Frame, the Yale professor, pointed out, math educators have yet to find a foolproof way to teach proofs—within the confines of a normal class schedule—to students who haven't gone to math camp or who don't "get it" on their own by reading the textbook. Roger Howe also has doubts that such a solution exists.

"One strength of mathematical culture is that it produces things that are extremely reliable, that can be used in lots of different ways and that stick around forever. Producing things like that requires a kind of culture and austerity that might never appeal to the masses. Perhaps math has to accept this lonely path," Howe said.

For a few months after dropping Math 230, I was content to leave math on its lonely path. But soon afterwards, I couldn't help getting jealous when I heard math majors speak reverentially of the "beauty" and "elegance" of their proofs. Some even talk about proof like it's a self-help technique.

"Math shapes my personality. By doing proofs, I learn how to be rigorous and careful," said Mu, the freshman Putnam fellow. "In a proof, you have to categorize a big problem into several possibilities, and you have to conceive of every possibility. In real life, when I'm faced with a difficulty and there are many directions I could go in, I consider each one, and then I can solve the problem." Mu said that he drafts yearlong plans, covering his academic and personal goals, and then writes weekly progress reports for himself.

The philosophical appeal of proof is what drew me, as a sophomore, to "Mathematical Logic," a philosophy course with an imposing title and a petite Korean instructor, Sun-Joo Shin. Professor Shin types up and hands out all the important proofs that she covers in lecture, so students don't have to scribble to keep up. She began the course by covering the bare essentials of a logical proof system and then built up from there, asking us to prove important theorems for homework along the way. The grand unifying theorem that we proved on the last day of class, Kurt Gödel's Completeness Theorem, was climactic and deliciously philosophical. Twenty-four mostly humanities-focused students and I walked away from the class with a proof background that most of us never would have gotten otherwise.

After that confidence-booster, I declared a major in mathematics and philosophy, and I dove back into the math department's upper-level courses, rejoining some of my former Math 230 classmates. I still struggle to follow lectures, and I'm not planning to take the Putnam exam anytime soon, but I'm getting a feel for mathematics, which, as the St. John's website points out, comes from the Greek word for "learning."

But maybe I'm just a math wimp for taking such a circuitous route back to proofs. Alex Brown, a freshman at Yale this year, started Math 230 last fall with only a few proofs from high school under his belt. The homework became overwhelming, he scored 20 points below the class average on the midterm, and he was ready to drop out. When he called his mom to break the news, something about her tone made him change his mind.

"She said, 'Okay, you can drop the class, but you've always wanted to do math, so this would be a good challenge for you.' It was like she was daring me to stick with the class, so I did. I busted my tail after the midterm, went to all of my professor's office hours and made sure I understood every homework problem."

Brown finished the class with a B+. This semester, he's taking more proof-based math classes, and he's gone to a few Math Society events. Even if he doesn't help Yale take on Harvard or MIT in the Putnam next year, Brown represents one more member of Yale's small math community, which needs him badly.

That, I can prove.