#### Yale University CTL Helmsley STEM Education Series

# Developing Mathematical Creativity: Physics Invention Tasks

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#### **Collaborative Principal Investigators**

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This work is supported by NSF DUE-1045227, NSF DUE-1045231, NSF DUE-1045250

## Why do you require physics?

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- Experimentation as a way of creating knowledge.

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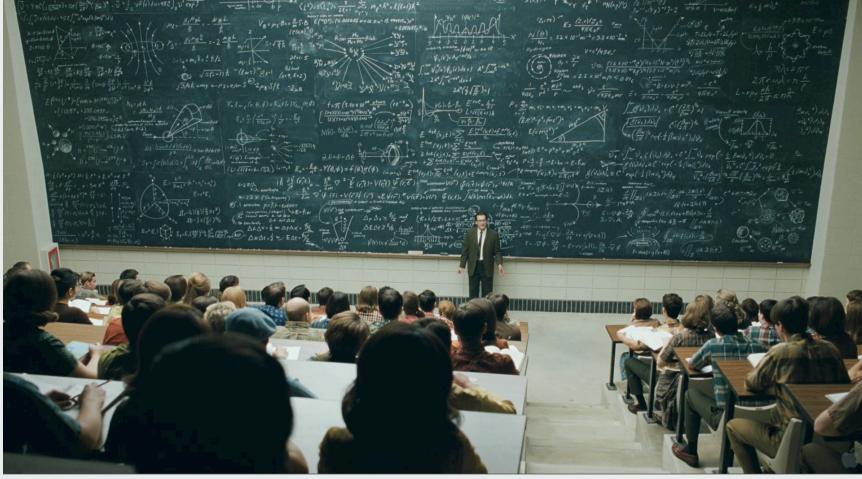
- Mathematization as a way of reasoning.
- Experimentation as a way of creating knowledge.

## Mathematization involves...

- representing ideas symbolically,
- defining problems quantitatively,
- producing solutions,
- and checking for coherence.

All in a coordinated effort to understand how the world works.

# Do students learn to mathematize through observation?



#### ...they learn recipes:

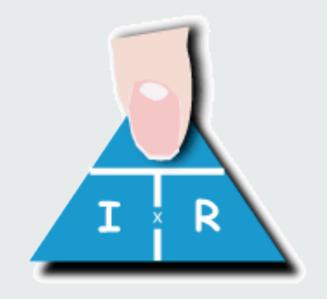
"There are many occasions when you have to use an equation in Science, particularly in Physics. The Equation Triangles are a way in which you can easily learn to use and rearrange equations, even if you are not confident in your Maths."



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# Affective measures reveal counterproductive practices

(from CLASS, 2006, 42 statement survey)

- When I solve a physics problem, I locate an equation that uses the variables given in the problem and plug in the values.
- I do not expect physics equations to help my understanding of the ideas; they are just for doing calculations.
- If I want to apply a method used for solving one physics problem to another problem, the problems must involve very similar situations.

## Affective measures: Learning Attitudes Surveys

- CLASS (Adams *et al.* 2006) U of Colorado, Boulder– typically average of ~10-15% drop in expert-like responses
- MPEX (1998) U of Md –showed systematic deterioration in expertise of student responses regarding the use of math in physics
- The deterioration is less severe in interactive engagement courses.

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#### Problems

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# How do successful students mathematize?



#### Features of successful problem solving

 Bing and Redish (2012) –interplay between formal mathematical manipulation and physical sense-making essential to success

Sherin (2001) - Engineering students (elite) in last physics course: flexible and generative understanding of equations is essential

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- Torigoe and Gladding (2012) –reasoning about symbolic representations correlates to course grades

## Mathematizing

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#### A generative use of mathematics is a hallmark of physics for which students have little preparation.

Our discipline has the potential to foster both.

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But do we?

# How do most students mathematize?

Obstacles

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### **Obstacles**



Concepts in the introductory course are well within a physicists' limits of mathematization, but are beyond or just at the edge of most students'.

Most instructors have forgotten what its like to struggle in this way.. have.

#### Rutgers study

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Consider the following statement about Winnie the Pooh's dream: *"There are three times as many heffalumps as woozles."* A correct equation to represent this statement, using *h* for the number of heffalumps and *w* for the number of woozles, is:

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3%	37%	2%	49%	9%	

 $\sigma_{\text{pooled}}=1.8\%$ 

p-value = 0.8418

#### **Rice Questions**

Bartholomew is making rice pudding using his grandmother's recipe. For three servings of pudding the ingredients include 0.75 pints of milk and 0.5 cups of rice. Bartholomew looks in his refrigerator and sees he has one pint of milk. Given that he wants to use all of the milk, which of the following expressions will help Bartholomew figure out how many cups of rice he should use?

0.5/0.75 0.75/0.5 0.5 x 0.75 (0.5 + 1) x 0.75 none of these

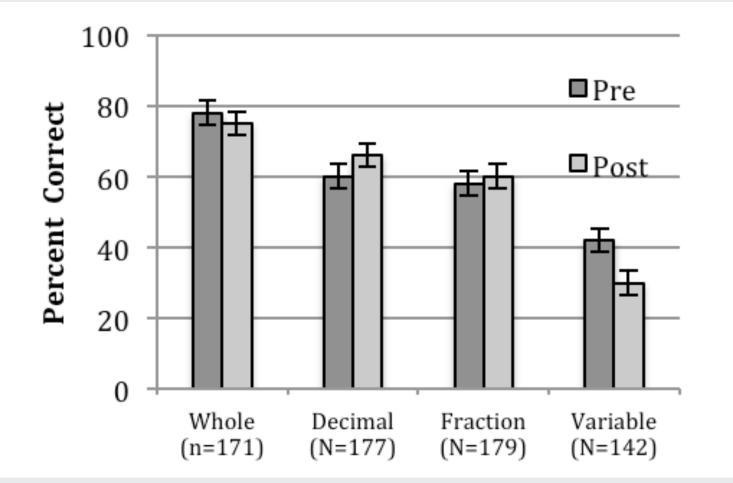
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### Numerical Complexity (Calculus-based Intro Mechanics)



	Top 20% (n <sub>sample</sub> =98)	The rest (n <sub>sample</sub> =363)	Effect size
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SAT_M	710	670	11.4
FCI % pre/change	+9	+9	
Math Reasoning % pre/ change	+4	-2	2.3/4.4
CLASS Problem Solving (Gen) % pre/change	-2	-10	
CLASS Personal Interest % pre/change	0	-9	

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Average of the Median MHI High School	Q	0.9* <i>Q</i>	5.5
		p-value < .02	

# NJ school math and socioeconomics (J. Anyon 1980)

MHI Quintile	Socioeconomic Status	Schoolwork culture
2 <sup>nd</sup>	Working class	Work is evaluated for obedience to procedure. Students learn to imitate the teacher in math class.
3 <sup>rd</sup> -4 <sup>th</sup>	Middle class	Work is getting the right answer. Creative activities are occasional, for fun but not part of learning. Students are given some choice in math on which of two procedures to use to get an answer.
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"The biggest obstacle to success is NOT limitation with math skills or knowing the definition of density...It's the institutional suppression of thinking."

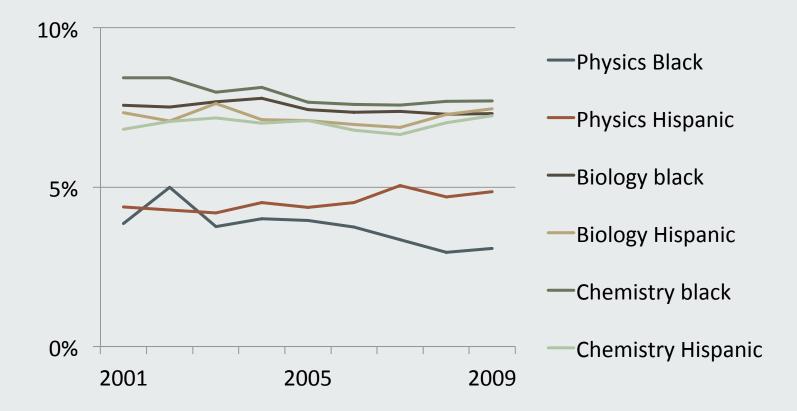
-Richard Steinberg 2011

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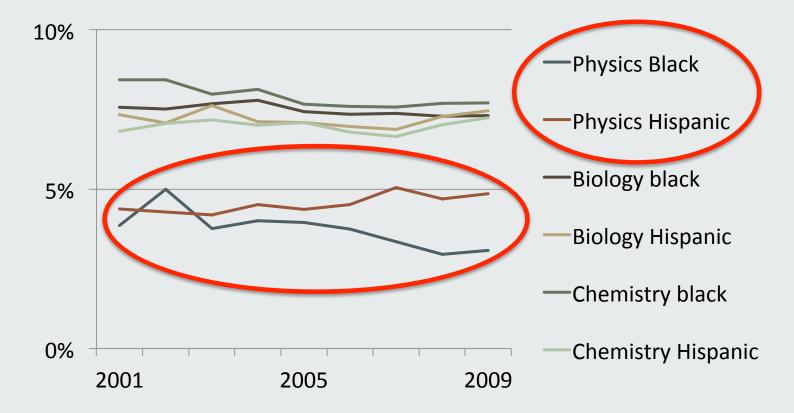
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# The percentage of the Bachelor's degrees granted to select underrepresented minorities\*

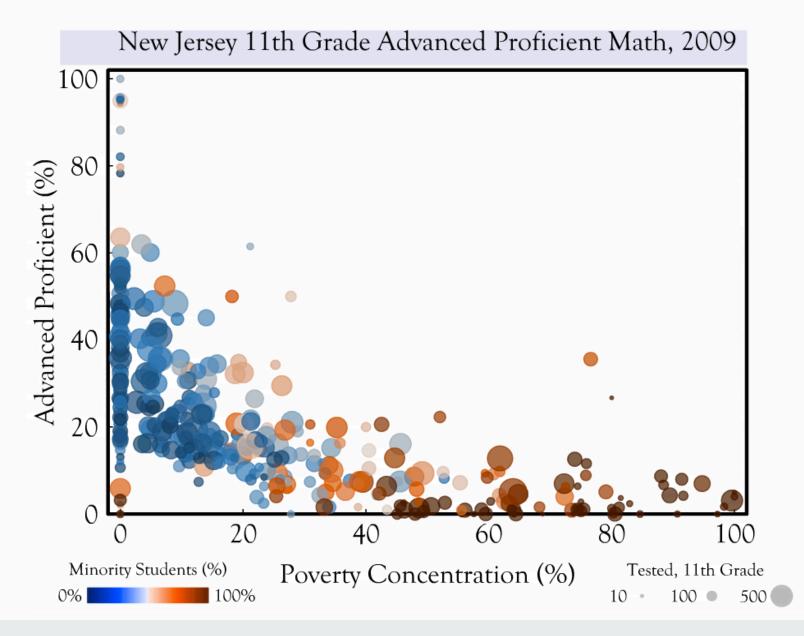


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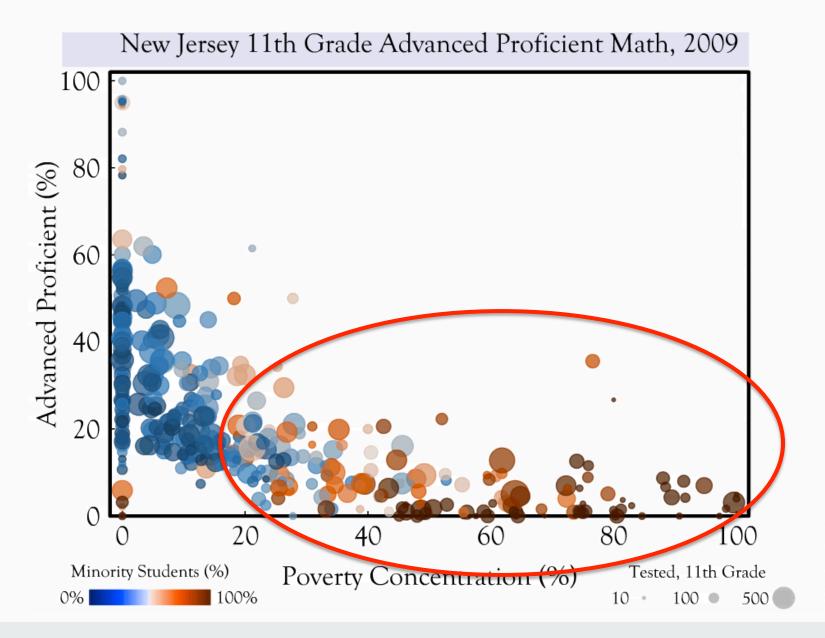
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Slide courtesy of Michael Marder



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#### Somebody else's problem

On the surface, this seems like a problem with prior math instruction. But it's not – math in physics has different goals than math in math.

Physics – flexible and generative mathematics in context Math – axiomatic reasoning in the absence of context

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  - What students master in their math courses is largely procedural.
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#### Problem

Most physics students, and especially students from low SES high schools, struggle to assimilate the habits of mind we model, and they leave our courses with even less expert-like mathematical attitudes and habits.



#### **Procedural Mastery**

+

#### **Conceptual Understanding**

#### **Procedural Mastery**

+

**Conceptual Understanding** 

### **Proceptual Understanding**

Find 47-35

- **Procedure:** Use number line, start at 47 count left 35 places
- Process(Flexibility): Start at 35, move to the right 12 places
- Proceptual (Generative): x=a-b represents the mathematical idea "difference"; and x=a-b implies that a=x+b

: Av=v<sub>f</sub>-v<sub>o</sub> therefore v<sub>f</sub>= $\Delta$ v + v<sub>o</sub>

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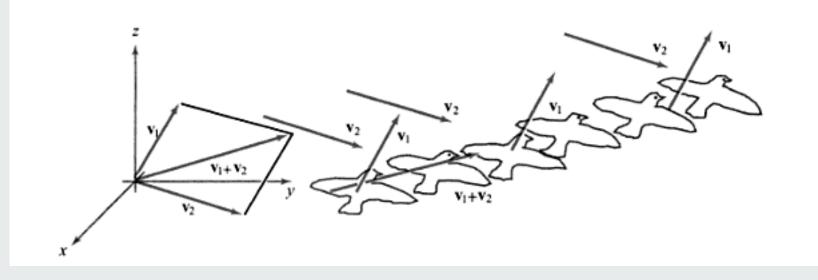
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comparison

### Proceptual divide

The mathematics of flexible procepts is easier than the mathematics of inflexible procedures. The gap is widening because the less successful are actually doing a qualitatively harder form of mathematics. (Tall 2008)

### **Proceptual physics**



relies on a tendency to seek invariance

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  - $\diamond$  Brahmia & Boudreaux (2016): Students errors can be traced to a failure to distinguish *products* from *factors* when reasoning about physics quantities. 72

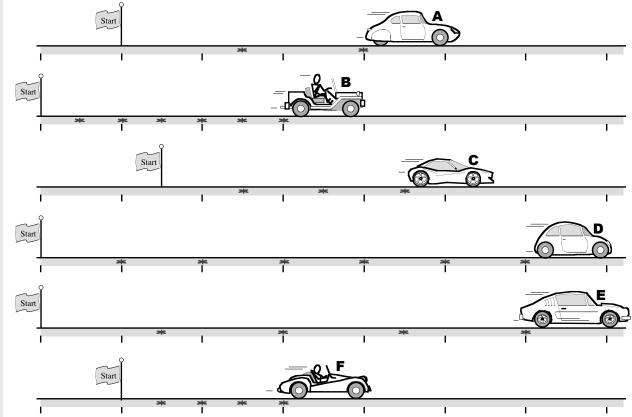
# Sample Invention Sequence 1

Your task this time is to come up with a **fastness index** for cars with dripping oil. All the cars drip oil once a second

## This task is a little harder than before.

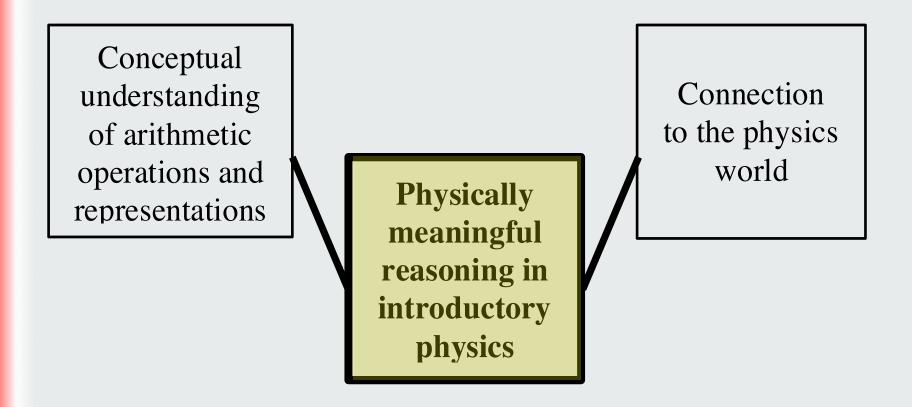
A company always makes its cars go the same fastness. We will not tell you how many companies there are.

You have to decide which cars are from the same company. They may look different!



## Quantification is a conceptual blend

*double scope arithmetic reasoning blend,* in which two distinct domains of thinking are merged to form a new cognitive space optimally suited for productive work

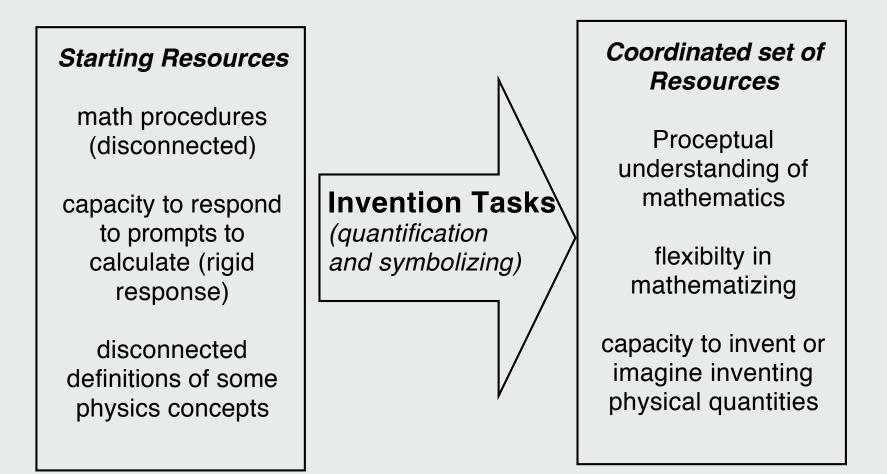


## ICC (Inventing with Contrasting Cases)

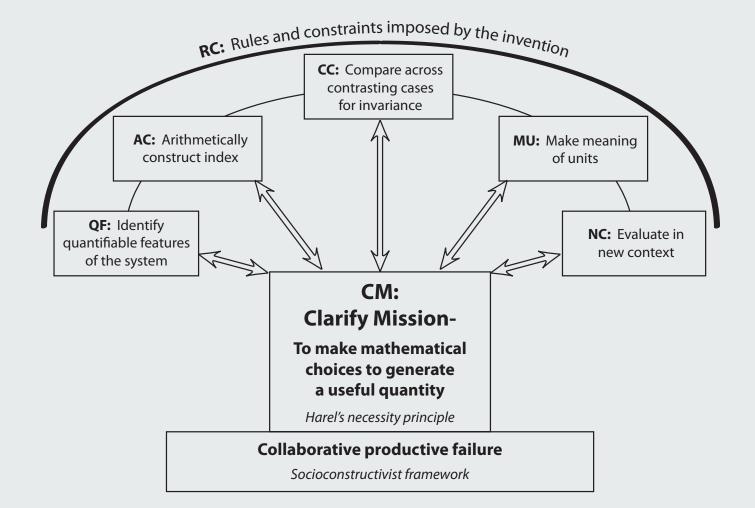
Schwartz, Chase, Oppezzo, & Chin 2011

- Instructional model designed to help students develop the tendency to
  - Seek invariance
  - Make sense with compound quantities
  - Contrasting helps students notice what matters and what doesn't
  - Preparation for subsequent instruction

## **Invention Instruction**

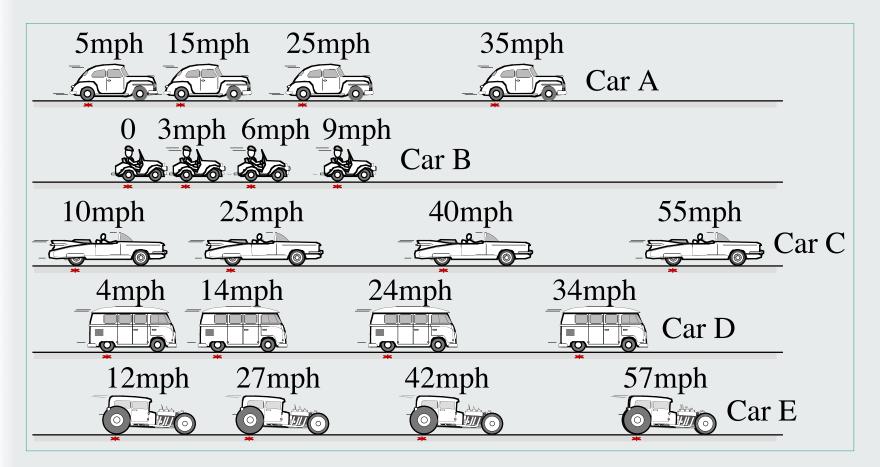


## **Applying ICC: Physics Invention Tasks**



# Sample Invention Sequence 1

These cars all drip oil once every second. Invent a <u>speeding-up</u> <u>index</u> that allows you to rank the cars in terms of how quickly they speed up.



# Sociocultural Benefits

- Valuing naïve understanding (Ross & Otero 2013)
- Shifting authority from instructor to social consensus (Ross & Otero 2013)

Addressing stereotype threat: Not remediation; students work, and struggle, collaboratively. (Steele) Developing self-efficacy: Invention process gives ownership of the knowledge to the student (Bandera, Sawtelle)

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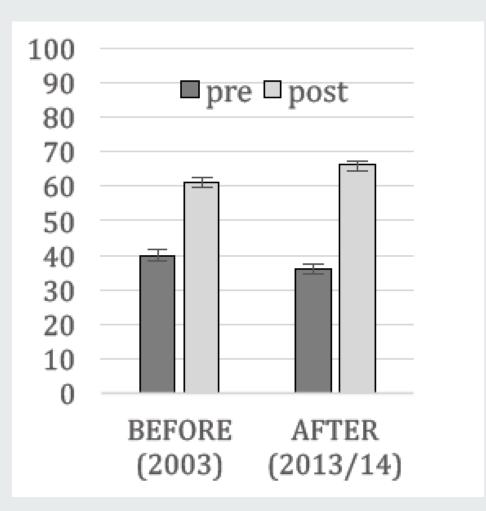
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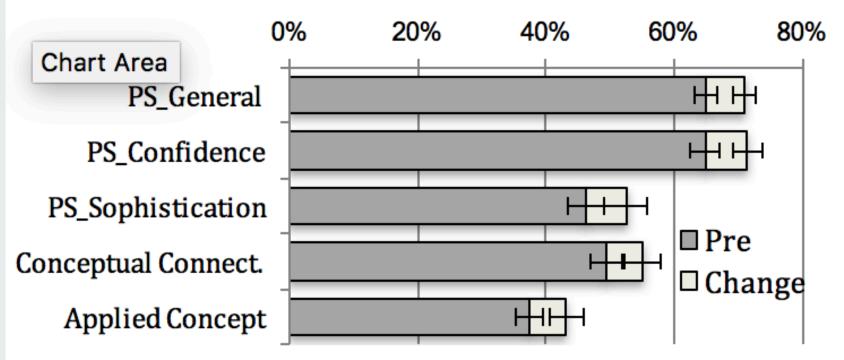
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#### FCI comparison (before the introduction of PITs, 2003, n=102 and after 2013/14, n=144



CLASS- physics categories associated with mathematical reasoning, pre-instruction and the gains over one semester.

> Combined Fall 2013 and Fall 2014, n=121. Error bars represent the standard error.



 Underprepared (precalc math placement) vs Mainstream (calculus math placement)

Simultaneous courses Same content, different curricula FCI, Math reasoning, CLASS and some MBL pre/post Fall 2013

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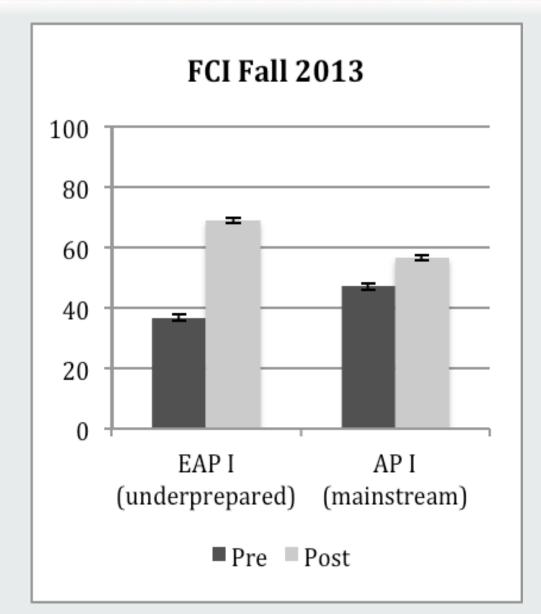
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## **Course Demographic Comparison**

	EAP I (Underprepared)	AP I (Mainstream)
# of students	~120	~700
Mean SAT	610	680
% URM	40%	12%
% female	30%	21%
Median MHI of sending district	0.7*Q p-value<.00000001	Q

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Force Concept Inventory;  $\sigma_{mean}$ : EAP I (n=135) 1.4%(pre), 1.5%(post); AP I (n=757) 0.8%(pre), 0.8%(post)

## CLASS

While the EAP course shows small positive gains, the AP course shows negative gains ~10% across PS categories.

## Mathematical Reasoning Item

A bicycle is equipped with an odometer to measure how far it travels. A cyclist rides the bicycle up a mountain road. When the odometer reading increases by 8 miles, the cyclist gains *H* vertical feet of elevation. Find an expression for the number of miles the odometer reading increases for every vertical foot of elevation gain.

$$\sin^{-1}\left(\frac{8}{H}\right)$$
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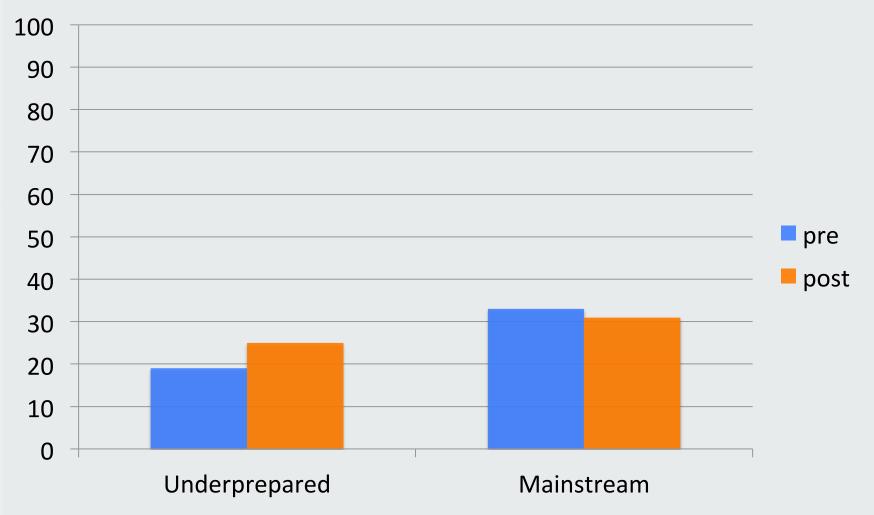
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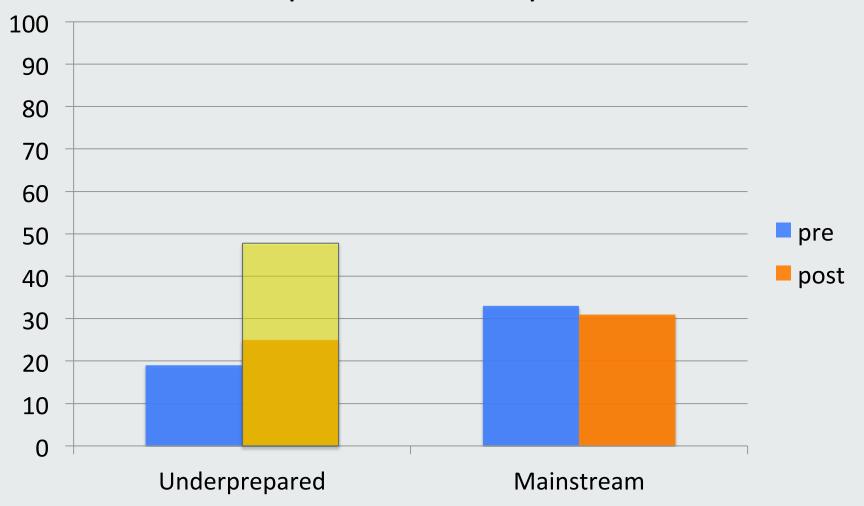
$$\sin^{-1}\left(\frac{8}{H}\right) \qquad \sin^{-1}\left(\frac{H}{8}\right) \qquad H/8 \qquad 8/H$$

None of these

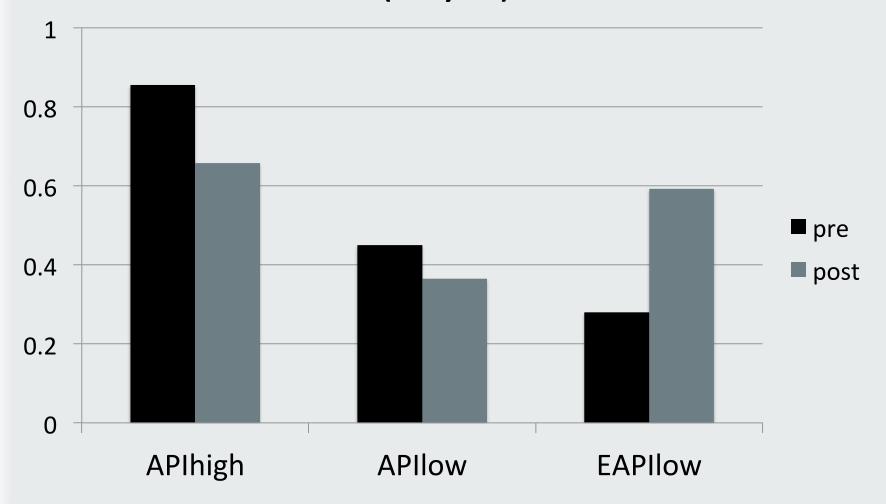
#### Bike Path RU Fall 2013 One semester of instruction



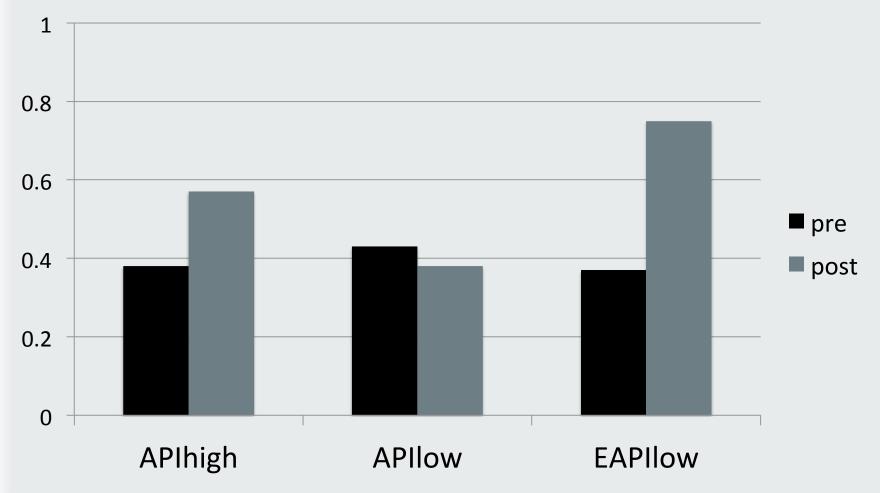
#### Bike Path RU (full year of instruction) $n_{115/6} = 187$ and $n_{123/4} = 583$



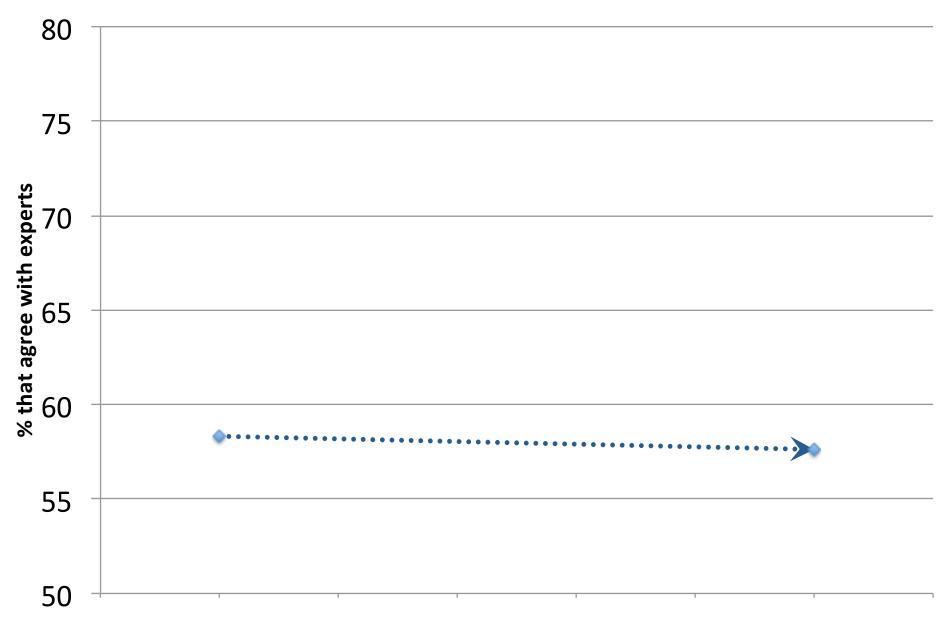
#### Rice Questions (SES) (full year)



#### Woozles(SES) (full year)

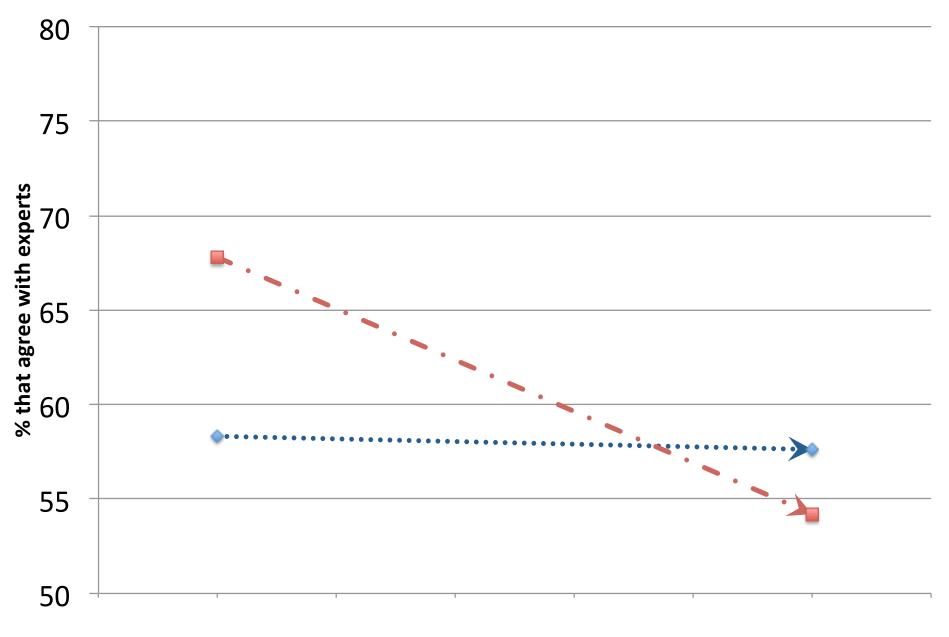


#### **CLASS Problem Solving - General**



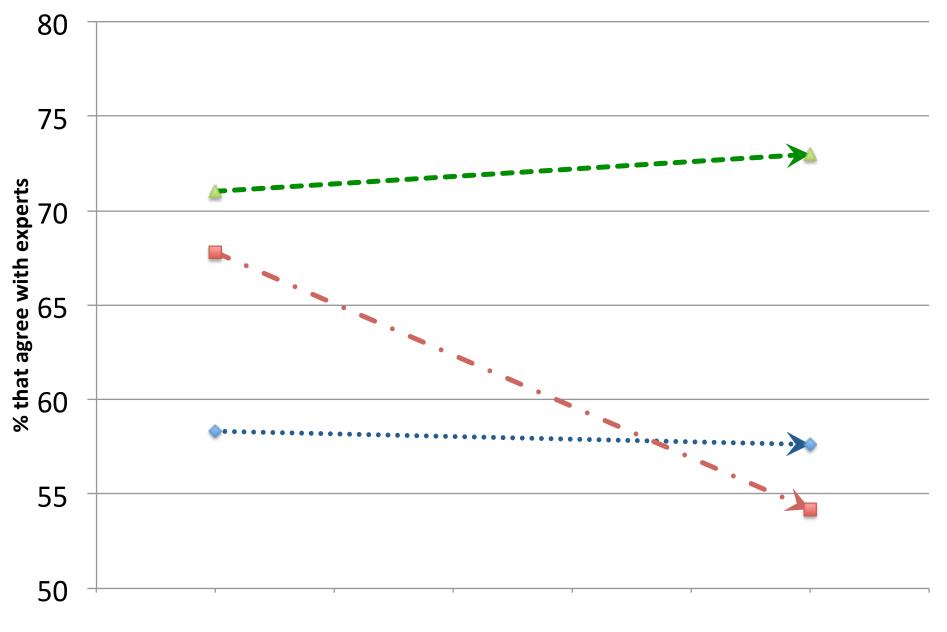


**CLASS Problem Solving - General** 



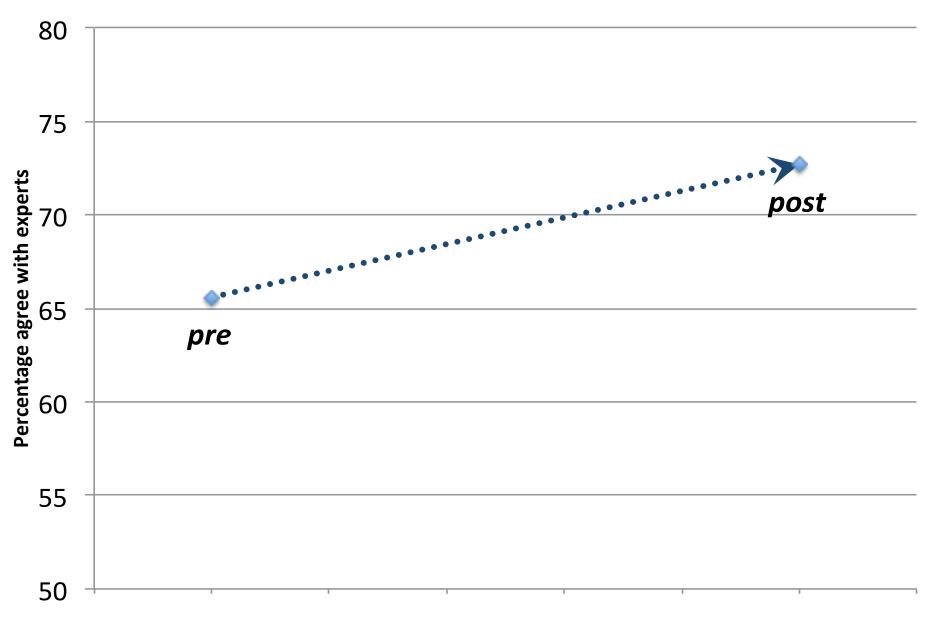


**CLASS Problem Solving - General** 



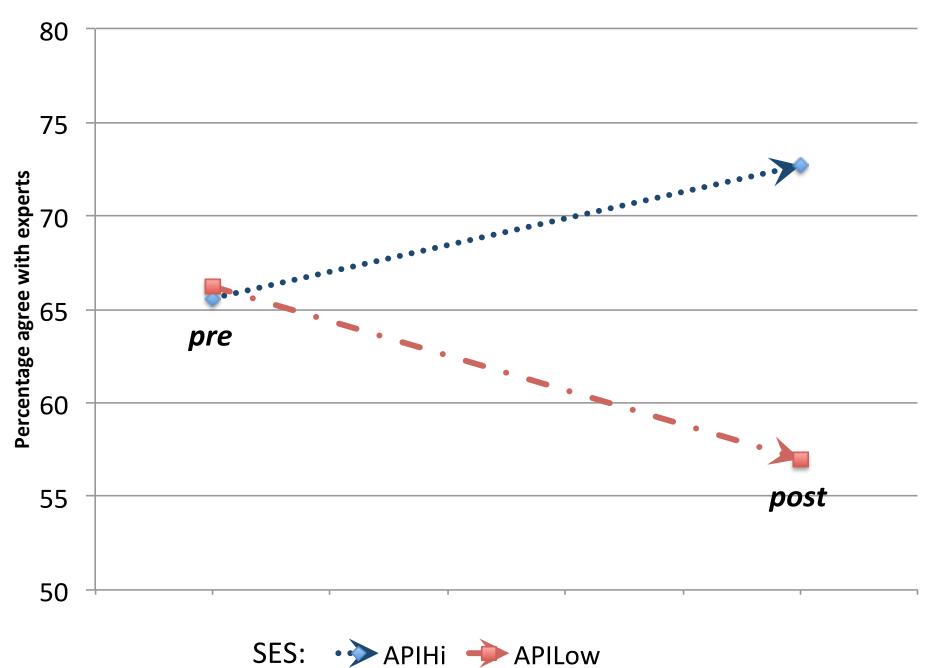
SES: •> APIHi + APILow + EAPILow

**CLASS Personal Interest** 

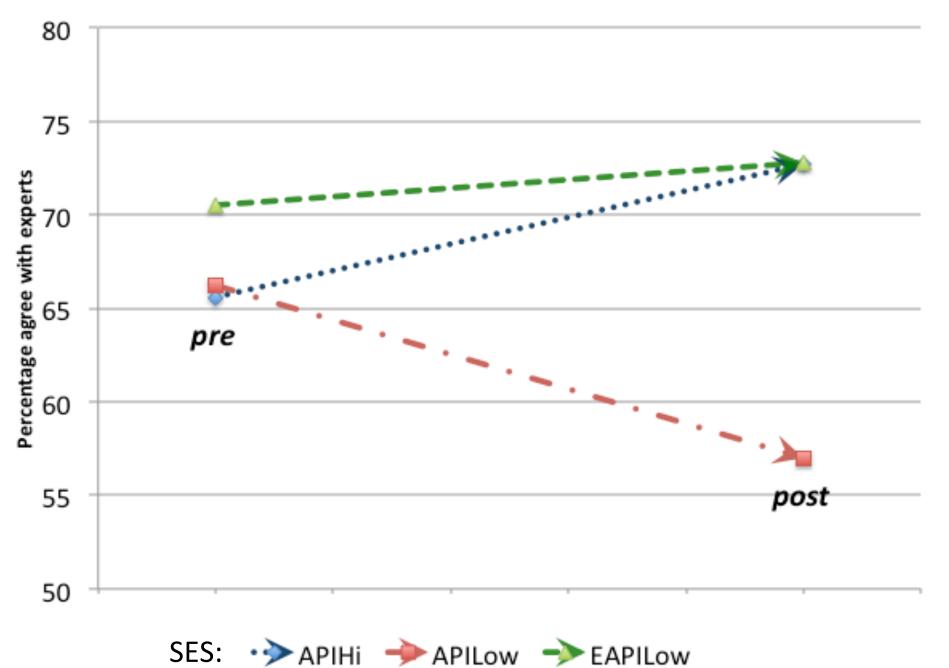




**CLASS Personal Interest** 



**CLASS Personal Interest** 



# Thank you!

#### **Physics Invention Tasks website:**

#### http://faculty.uw.edu/pits

#### Password (case sensitive): Treehouse